Inżynierska metoda obliczania krytycznej siły ściskającej pudła z tektury litej

Engineering calculation procedure of critical compressive force of paperboard packages

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W artykule przedstawiono badania stateczności opakowań z tektury litej pod wpływem kompresji. Został zaproponowany empiryczny matematyczny model opisujący obliczenia krytycznej siły ściskającej pudła z tektury litej. Utworzony model matematyczny uwzględnia wpływ wysokości pudelka, która oddziaływuje również na krytyczną siłę kompresji. Proponowany model matematyczny pozwala na optymalizację właściwości wytrzymałości opakowań tekturowych. Podano porównanie uzyskanych obliczeń siły krytycznej z wynikami eksperymentalnymi. Wykazano dostateczną zbieżność obu wyników teoretycznych i eksperymentalnych. Proponowany inżynierny matematyczny model obliczeniowy może być stosowany do projektowania prostopadłościennego pudła z tektury litej.
Słowa kluczowe: *tektura, opakowanie, ściskanie, siła krytyczna, nowy wzór empiryczny*

This paper presents a testing of paperboard packages stability under compression. Empirical mathematical model has been proposed describing the calculations of critical compression force of rectangular – parallelpiped paperboard packages. The created mathematical model evaluates the effect of package height that also affects the critical compression force. This model allows to optimize the strength characteristics of paperboard packages. A comparison of obtained calculations of critical force with the experimentally determined one is given. It has shown a sufficient coincidence of both theoretical and experimental results. Proposed mathematical engineering calculation model can be applied for the design of rectangular – parallelepiped paperboard packages.
Keywords: *paperboard, package, compression, critical force, new empirical formula*

Introduction

In terms of consumed quantity, paper and paperboard have been for a long time the main packing materials for different products and goods. The market volume of this packaging makes up about 48.8% of the whole packaging used in the European Union (from 41.7% in France to 59.6% in Sweden). Although plastic industry is developing rapidly, the usage of paperboard as a cheap and ecological packing material is not decreasing. Paperboard is made from renewable resources and the paperboard-based materials decompose relatively easy under the effect of humidity and usual atmospheric conditions [2, 8].

Packaging construction and design is being constantly improved. Complex problems have to be solved for ensuring package reliability during distribution and transportation of packed goods, since the packed products may be damaged by the developing static and dynamic forces. Static load of the packaging is usually caused by internal pressure of the product and the impact of other packed goods lying above. The most popular paperboard packages are boxes in the shape of parallelepiped rectangles.

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Following the requirements of European Union directives, the goal in further development of packaging is manufacturing durable packaging and minimizing the amount of materials needed for that [1].

One of the main characteristics of packaging is its resistance to vertical load which increases significantly, for instance, by stacking packed goods on pallets and storing. In this case, the highest loads act on the goods lying below, and the acting forces are directed vertically downwards. The packed product can take a part of the load and thus reduce the danger of damaging the package however it is not always sufficient.

An important step in designing the packages most suitable for packaging is to understand how static forces affect the paperboard package. Early works of McKee [12] and Grångard [6] were based on empirical dependencies in predicting the paperboard packaging resistance to compression. The papers mentioned above lack studies concerning the packaging resistance to compression, with regard to the paperboard direction.

In order to predict the compression strength of boxes, several authors, e.g. McKee et al. [12] and Urbanik [17], proposed various approaches. These approaches were based on criteria that enable prediction of the buckling mode *m* of buckled box panels'. Two of the proposed criteria by Urbanik and Frank [17] appear to lead to compression strength predictions on the basis of the large set of experimental data. The first assumption is that the side panels and the top and end panels buckle independently into a number of half-waves, which corresponds to the weakest critical buckling load that can be theoretically predicted for each panel. The second assumption is that the buckling mode *m*, which yields the lowest critical buckling load for the side panel, has to be applied to both panels. Viguié et al. [18] investigated the dependency of the compression strength and the buckling mode of boxes with respect to their geometry and loading conditions. Both monotonic and cyclic loading conditions were performed.

Buckling strength of simply supported corrugated board panels subjected to edge compressive loading has been studied experimentally using a specially developed test equipment [7]. It should be also noted that the review of many studies in this field is presented in the paper [11].

In paper [15], a simple engineering model for prediction of package collapse load has been proposed. The model is based on the concept of dividing the package into panels and corner panels.The model predictions are in close agreement with a very broad class of packages, board qualities and loading directions.

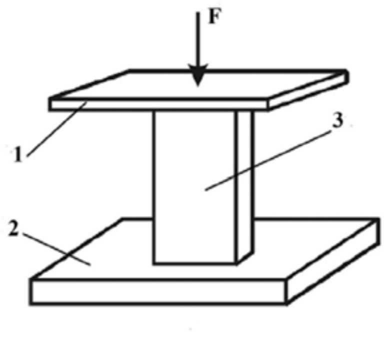

Fig. 1. Compression testing scheme of a package under the action of vertical force *F*, N: 1 – moving base support; 2 – bottom base; 3 – package under compression

Review [3] focuses on the process of box compression and the utility of box compression testing, bringing previous work back to the fore to provide useful background for current studies. It examines the conditioning and testing process in detail, discusses the state of the art in compression estimation, and explores various parameters that affect box compression strength that are not captured in most current industry models.

The crippling method has been shown in paper [10] to estimate the compression strength of milk and cigarette boxes utilizing a total of four different materials and five different load cases.

In paper [9], a new approach of the BCT index determination procedure is presented. This procedure is based on the multi-layered orthotropic plates theory. The numerical simulations of the BCT test have been carried out to compare with analytical results.

In papers [4, 5], authors present numerical analysis of the critical force determination, causing the loss of stability of multilayered orthotropic plates section with the periodic core.

The analysis of the available research publications leads to a conclusion that there is a lack of publications in which the engineering calculation procedure of optimal geometrical parameters of rectangular – parallelepiped paperboard packages would be proposed, especially evaluating the effect of package height.

The goal of this work is to develop the empirical mathematical model that might be used for box compression strength estimation and to compare calculation results with the set of experimental data and to evaluate the applicability of the model to the engineering calculations.

Thus the aim of present work is to propose the engineering calculation procedure of critical compression force of rectangular – parallelpiped paperboard packages allowing the choice optimal geometrical parameters of the package.

Table 1. Comparison of paperboard technical characteristics [8]

No.	Type of paperboard	Grammage, ISO 536 g/m²	Thickness, ISO 534 µm	D = flexural stiffnesses L&W[1], (5°), mNm ISO 5628		ECT = edge crush test, SCT, kN/m ISO 9895	
				MD[2]	*CD*[3]	*MD*[2]	*CD*[3]
1	**2**	**3**	**4**	**5**	**6**	**7**	**8**
1	MC Mirabell	400	565	60.9	24.4	12.7	8.5
2	MC Mirabell	320	435	31.8	13.3	9.8	7.4
3	Kromopak	300	430	34.3	14.3	9.2	6.8
4	Kromopak	275	395	29.0	12.0	8.6	6.2
5	Korsnäs Carry	400	585	113.0	55.3	11.2	8.4
6	Korsnäs Light	290	420	41.9	21.2	8.5	6.1

[1] L&W device-measured moment needed for bending the sample material to an angle of 5°
[2] MD – machine direction
[3] CD – cross machine direction

Experimental method and materials

Different types of paperboard package constructions were used for the creation of engineering calculation procedure of critical compression force. The samples of testing object are shown in Figs 1 and 2. The packages of such sizes are widely used for packing grain products (rice, buckwheat, etc.). The choice of this type of packaging specimens was determined by their wide usage for food products packing.

Considering the potential conditions of storing, transportation and maintenance, the packages were made of different paperboard types: soft MC Mirabell paperboard, medium soft Kromopak paperboard and tough paperboard Korsnäs Carry and Korsnäs Light. The technical characteristics provided by the manufacturers of these paperboards are listed in Table 1.

The geometrical parameters of packages are shown in Fig. 2 and listed in Table 2.

The findings of proposed calculation procedures were later compared with the findings of experimental compression tests (see Fig. 3 and Table 2 (Column 5)) that were analyzed in paper [8].

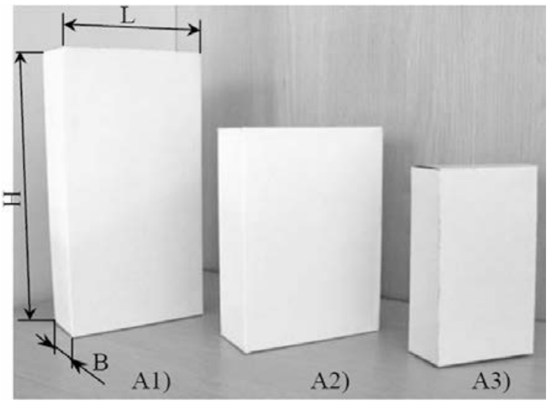

Fig. 2. The specimens of tested packages A1) box size I (H = 230 mm, L = 118 mm, B = 48 mm), A2) box size II (H = 165 mm, L = 118 mm, B = 48 mm), A3) box size III (H = 137 mm, L = 77 mm, B = 37 mm)

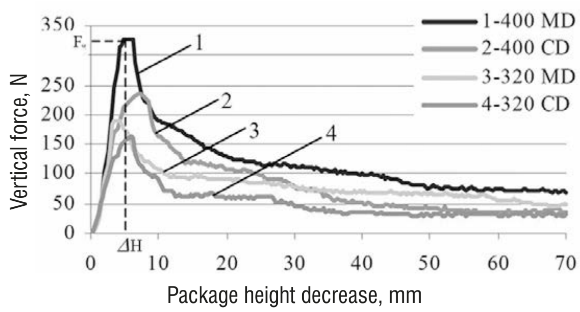

Fig. 3. Graph of resistance to compression of A1 boxes made from MC Mirabell paperboard: *1* – 400 g/m² paperboard (machine direction); *2* – 400 g/m² paperboard (cross-machine direction); *3* – 320 g/m² paperboard (machine direction); *4* – 320 g/m² paperboard (cross-machine direction); F_{cr} – critical compression force, ΔH – package height decrease at critical compression force, mm [8]

Modelling of the critical force

McKee's formula [12] is often applied for calculation of critical compression force of paperboard packages with geometrical parameters $L \times B \times H$:

$$F_{cr} = aECT^{b}\left(\sqrt{D_x D_y}\right)^{1-b} P^{2b-1}, \; H/P > 0.143 \qquad (1)$$

or a simplified version

$$F_{cr} = 5.876 \cdot ECT\sqrt{h \cdot P} \qquad (2)$$

where F_{cr} – box compression force N; D_x, D_y – flexural stiffnesses, Nm (parallel to x and perpendicular to y axis); ECT – edgewise compression test; $P=2L+2B$ – perimeter of the box, m; H – height of the box, m; h – thickness of the paperboard, m.

Table 2. Experimental data of paperboard package compression tests (23°C, relative humidity RH=50%). Each value (Column 5, 7) is the average of 3-4 tests

	Dimensions mm	Type of paperboard, No.	Technical characteristics of paperboard	$F_{cr,exp}$, N	$F_{cr,cal}$, N (16)	ΔH, mm
1	2	3	4	5	6	7
1	box size I	1	$D_x=60.9$; $D_y=24.4$; $SCT_x=12.7$; $SCT_y=8.5$;	328.58	248.02	5.60
2	box size I	1	$D_x=24.4$; $D_y=60.9$; $SCT_x=8.5$; $SCT_y=12.7$;	235.47	204.01	7.00
3	box size I	2	$D_x=31.8$; $D_y=13.3$; $SCT_x=9.8$; $SCT_y=7.4$;	190.96	172.37	3.50
4	box size I	2	$D_x=13.3$; $D_y=31.8$; $SCT_x=7.4$; $SCT_y=9.8$;	164.34	143.09	5.95
5	box size II	1	$D_x=60.9$; $D_y=24.4$; $SCT_x=12.7$; $SCT_y=8.5$;	320.18	268.05	4.20
6	box size II	1	$D_x=24.4$; $D_y=60.9$; $SCT_x=8.5$; $SCT_y=12.7$;	252.64	220.50	5.60
7	box size II	2	$D_x=31.8$; $D_y=13.3$; $SCT_x=9.8$; $SCT_y=7.4$;	210.99	186.29	4.90
8	box size II	2	$D_x=13.3$; $D_y=31.8$; $SCT_x=7.4$; $SCT_y=9.8$;	166.79	154.65	4.55
9	box size III	1	$D_x=60.9$; $D_y=24.4$; $SCT_x=12.7$; $SCT_y=8.5$;	267.35	256.21	5.60
10	box size III	1	$D_x=24.4$; $D_y=60.9$; $SCT_x=8.5$; $SCT_y=12.7$;	230.18	210.75	4.55
11	box size III	2	$D_x=31.8$; $D_y=13.3$; $SCT_x=9.8$; $SCT_y=7.4$;	210.04	178.06	3.15
12	box size III	2	$D_x=13.3$; $D_y=31.8$; $SCT_x=7.4$; $SCT_y=9.8$;	160.98	147.82	4.90
13	box size I	3	$D_x=34.3$; $D_y=14.3$; $SCT_x=9.2$; $SCT_y=6.8$;	183.96	187.91	4.20
14	box size I	3	$D_x=14.3$; $D_y=34.3$; $SCT_x=6.8$; $SCT_y=9.2$;	164.88	155.89	5.25
15	box size I	4	$D_x=29.0$; $D_y=12.0$; $SCT_x=8.6$; $SCT_y=6.2$;	161.88	172.86	3.85
16	box size I	4	$D_x=12.0$; $D_y=29.0$; $SCT_x=6.2$; $SCT_y=8.6$;	132.45	143.18	4.20
17	box size II	3	$D_x=34.3$; $D_y=14.3$; $SCT_x=9.2$; $SCT_y=6.8$;	196.22	203.09	3.15
18	box size II	3	$D_x=14.3$; $D_y=34.3$; $SCT_x=6.8$; $SCT_y=9.2$;	183.96	168.48	4.90
19	box size II	4	$D_x=29.0$; $D_y=12.0$; $SCT_x=8.6$; $SCT_y=6.2$;	176.60	186.83	2.80
20	box size II	4	$D_x=12.0$; $D_y=29.0$; $SCT_x=6.2$; $SCT_y=8.6$;	149.62	154.74	3.50
21	box size III	3	$D_x=34.3$; $D_y=14.3$; $SCT_x=9.2$; $SCT_y=6.8$;	196.22	194.12	3.85
22	box size III	3	$D_x=14.3$; $D_y=34.3$; $SCT_x=6.8$; $SCT_y=9.2$;	166.79	161.04	3.15
23	box size III	4	$D_x=29.0$; $D_y=12.0$; $SCT_x=8.6$; $SCT_y=6.2$;	186.41	178.57	3.15
24	box size III	4	$D_x=12.0$; $D_y=29.0$; $SCT_x=6.2$; $SCT_y=8.6$;	142.36	147.91	3.50
25	box size I	5	$D_x=113.0$; $D_y=55.3$; $SCT_x=11.2$; $SCT_y=8.4$;	446.88	480.35	5.95
26	box size I	5	$D_x=55.3$; $D_y=113.0$; $SCT_x=8.4$; $SCT_y=11.2$;	365.75	412.37	5.25
27	box size II	5	$D_x=113.0$; $D_y=55.3$; $SCT_x=11.2$; $SCT_y=8.4$;	456.22	519.15	5.25
28	box size II	5	$D_x=55.3$; $D_y=113.0$; $SCT_x=8.4$; $SCT_y=11.2$;	370.39	445.68	5.60
29	box size III	5	$D_x=113.0$; $D_y=55.3$; $SCT_x=11.2$; $SCT_y=8.4$;	394.90	496.22	5.25
30	box size III	5	$D_x=55.3$; $D_y=113.0$; $SCT_x=8.4$; $SCT_y=11.2$;	360.01	425.99	5.60
31	box size I	6	$D_x=41.9$; $D_y=21.2$; $SCT_x=8.5$; $SCT_y=6.1$;	197.91	253.77	3.85
32	box size I	6	$D_x=21.2$; $D_y=41.9$; $SCT_x=6.1$; $SCT_y=8.5$;	195.38	219.41	3.85
33	box size II	6	$D_x=41.9$; $D_y=21.2$; $SCT_x=8.5$; $SCT_y=6.1$;	246.10	274.27	5.25
34	box size II	6	$D_x=21.2$; $D_y=41.9$; $SCT_x=6.1$; $SCT_y=8.5$;	224.49	237.14	5.95
35	box size III	6	$D_x=41.9$; $D_y=21.2$; $SCT_x=8.5$; $SCT_y=6.1$;	262.54	262.16	4.90
36	box size III	6	$D_x=21.2$; $D_y=41.9$; $SCT_x=6.1$; $SCT_y=8.5$;	218.53	226.66	3.85

A popular technique of *BCT* (box compression strenght) evaluation for corrugated board was created based on empirical data. Different values of non-dimensional empirical constants *a* and *b* can be found in literature for packages made of corrugated board: $a = 1.97$, $b = 0.753$ [6] and $a = 2.05$, $b = 0.746$ [3].

Due to high inaccuracy, the calculations obtained according to the formulas (1) and (2) for paperboard packages can not be compared with experimental data.

But the dependence of the box height is not evaluated in formulas (1) and (2). The experimental data presented in this paper show that *BCT* depends on height of package (see experimental data of the same material box size I and box size II in Table 2). Thus the new empirical formula is developed in this paper.

The structural formula of critical force. A big number of factors describing the process of stability loss and their complex interaction require a new concept for developing and analyzing a model, compared to the traditional methods of analyzing separate elements of such packages. Such a concept could be the theoretical-experimental multi-factor analysis done by the author [10]. The present paper considers its application in solving the stability problems of packages (rectangular boxes) in order to achieve adequate modelling of their deformation and stability.

Taking into account the papers mentioned above [4 - 6, 9, 13, 14, 16], we have developed following structural equation of critical compressive force for packages of different sizes and different orthotropic directions under axial force:

$$F_{cr} = K_c \chi(t_L, t_B, t_H, t_D) = K_c \chi\left(\frac{L}{H}, \frac{B}{H}, \frac{H}{h}, \frac{D_x}{D_y}\right),$$

$$K_c = \frac{\pi^2 \sqrt{D_x D_y}}{H} \tag{3}$$

$t_L = L/H$ – parameter of the package length; $t_B = B/H$ - parameter of the package width; $t_H = H/h$ – parameter of the paperboard thickness; $t_D = D_x/D_y$ – relative rigidity in machine and cross-machine direction; $\chi(t_L, t_B, t_H, t_D)$ – correcting function, determined experimentally regarding to the total loss of stability.

Let us use the experimental findings from Table 2 for the correcting function included in the structural formula (3). Four independent variables were chosen as non-dimensional parameters: $t_L = L/H$, $t_B = B/H$, $t_H = H/h$, $t_D = D_x/D_y$.

In accordance with Equation (3), we obtain the following mathematical model:

$$\chi = \frac{F_{cr}}{K_c} = C_0 \left(\frac{L}{H}\right)^{A_1} \left(\frac{B}{H}\right)^{B_1} \left(\frac{H}{h}\right)^{C_1} \left(\frac{D_x}{D_y}\right)^{D_1} \tag{4}$$

The expression can be written as follows:

$$\ln\left(\frac{F_{cr}}{K_c}\right) = \ln C_0 + A_1 \ln\left(\frac{L}{H}\right) + B_1 \ln\left(\frac{B}{H}\right) + C_1 \ln\left(\frac{H}{h}\right) + D_1 \ln\left(\frac{D_x}{D_y}\right) \tag{5}$$

or

$$\hat{y} = b_0 + b_1 x_1 + b_2 x_2 + b_3 x_3 + b_4 x_4 \tag{6}$$

where

$$\hat{y} = \ln\left(\frac{F_{cr}}{K_c}\right), \ x_1 = \ln\left(\frac{L}{H}\right), \ x_2 = \ln\left(\frac{B}{H}\right),$$
$$x_3 = \ln\left(\frac{H}{h}\right), \ x_4 = \ln\left(\frac{D_x}{D_y}\right) \tag{7}$$

$$b_0 = \ln C_0, \ b_1 = A_1, \ b_2 = B_1, \ b_3 = C_1, \ b_4 = D_1. \tag{8}$$

Calculation of coefficients of multiple linear regression. Let us present the measurement data and the coefficients of the model in a matrix form:

$$\mathbf{y} = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_n \end{bmatrix}, \ \mathbf{X} = \begin{bmatrix} 1 & x_{11} & x_{12} & \cdots & x_{1m} \\ 1 & x_{21} & x_{22} & \cdots & x_{2m} \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 1 & x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}, \tag{9}$$
$$\mathbf{b} = \begin{bmatrix} b_0 \\ b_1 \\ \vdots \\ b_m \end{bmatrix}, \ \mathbf{e} = \begin{bmatrix} e_1 \\ e_2 \\ \vdots \\ e_n \end{bmatrix}$$

where $\mathbf{y}$ is the measurement vector-column for measuring the critical force, $y_i = \ln\left(F^i_{cr,exp}/K_c\right)$ (observed values of the dependent variable); $\mathbf{X}$ – dimension matrix $n \times (m+1)$, in which the *i*-th row $i = 1,2,...,n$ represents the *i*-th observation of the vector of independent variable values $x_1, x_2, ..., x_m$, values corresponding to the variables at given free term b_0; $\mathbf{b}$ – vector-column of dimension $(m + 1)$ parameters of multiple regression equation; $\mathbf{e}$ – vector-column of dimension *n* of deviations $e_i = y_i - \hat{y}_i$ where y_i depends on $\hat{y}_i$ obtained from the regression equation:

$$\hat{y}_i = b_0 + b_1 x_{i1} + ... + b_m x_{im}, \ i = 1,2,...,n, \ \hat{\mathbf{y}} = \mathbf{Xb} \tag{10}$$

The matrix form of the relation is:

$$\mathbf{e} = \mathbf{y} - \mathbf{Xb} \tag{11}$$

According to the least squares method:

$$\sum_{i=1}^{n} e_i^2 = \mathbf{e}^T \mathbf{e} = (\mathbf{y} - \mathbf{Xb})^T (\mathbf{y} - \mathbf{Xb}) \to \min \tag{12}$$

where $\mathbf{e}^T = (e_1, e_2, ..., e_n)$, i.e., the superscript *T* means a transponent matrix. It may be shown that the previous condition is fulfilled if the vector-column of coefficients $\mathbf{b}$ can be obtained by the following formula:

$$\mathbf{b} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \mathbf{y} \tag{13}$$

where $\mathbf{X}^T$ is a matrix transposed to matrix $\mathbf{X}$, and $(\mathbf{X}^T \mathbf{X})^{-1}$ is a matrix inverse to $(\mathbf{X}^T \mathbf{X})$. The relation is valid for equations of regression with a random number *m* of explanatory variables.

To ensure the quality of the model, it is necessary to have $n > 3m$, where *n* is the number of observations and *m* is the number of factors.

The model of multiple regression is evaluated by using determination coefficient R^2, *R* is the multiple coefficient of correlation between the dependent variable and the explanatory parameters:

$$R^2 = 1 - \frac{\sum_{i=1}^{n}(y_i - \hat{y}_i)^2}{\sum_{i=1}^{n}(y_i - \bar{y})^2} \quad (14)$$

where the average value of the dependent variable

$$\bar{y} = \frac{1}{n}\sum_{i=1}^{n} y_i \quad (15)$$

Model comprises multiplier K_C, which is included in Formula (3), and correcting function χ. It is assumed that the function depends on the set of relevant non-dimensional parameters. Model is based on experiments with different mechanical and geometrical dimensions of packages (36 different boxes). The model will also be valid for boxes with other parameters that are within the parameter range studied in the present paper.

Results and discussion

The experimental findings and the parameters of the boxes under testing are presented in Table 2. The experimentally obtained values of the critical force of compression $F_{cr,exp}$ are shown in column 5 of Table 2. Kolumn 7 presents the corresponding values ΔH of package height decrease.

The analysis of the data presented in Table 2 allows us to determine the areas of changes in the non-dimensional parameters: $t_L \in [0.33; 0.86]$; $t_B \in [0.16; 0.35]$; $t_H \in [234.2; 582.3]$; $t_D \in [0.11; 9.42]$.

In our case, $n = 36$, $m = 4$. Having the findings of the experiment, we can evaluate the coefficients of linear regression (10) by using the least squares method:

$b_0 = -0.968$, $b_1 = 0.122$, $b_2 = 0.141$, $b_3 = 1.029$, $b_4 = 0.107$.

The multiple coefficient of correlation (14) between the dependent variable and the explanatory parameters is equal to: $R = 0.912$.

Finally, the following mathematical model was developed upon the basis of the experimental findings:

$$F_{cr,cal} = \frac{\pi^2\sqrt{D_x D_y}}{H} 0.380 \left(\frac{L}{H}\right)^{0.122} \left(\frac{B}{H}\right)^{0.141} \left(\frac{H}{h}\right)^{1.029} \left(\frac{D_x}{D_y}\right)^{0.107}$$

$$R^2 = 0.821 \quad (16)$$

Also knowing the critical compression force $F_{cr,cal}$ from Equation (16) we can calculate important geometrical parameter of the package – height:

$$H = \left[0.380 \frac{\pi^2\sqrt{D_x D_y}}{F_{cr,cal} h^{1.029}} L^{0.122} B^{0.141} \left(\frac{D_x}{D_y}\right)^{0.107}\right]^{4.2735} \quad (17)$$

Note that the correlations given by Eqs (16) and (17) are valid only within the intervals of t_L, t_B, t_H, and t_D for which the data were obtained experimentally.

The coefficient of multiple determination R^2 in (16) was calculated by Equation (14) for the critical force of compression. The results are presented in Table 2 (column 6).

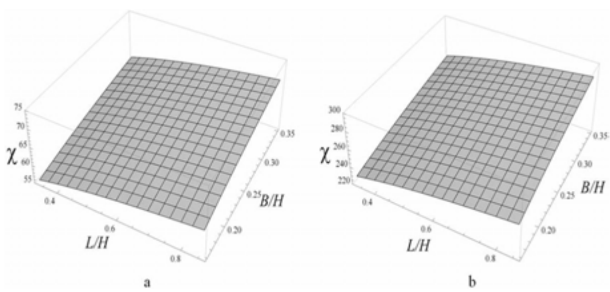

Fig. 4. Dependence of non-dimensional critical force $\chi = F_{cr,cal}/K_c$ on $t_L = L/H$ and $t_B = B/H$: a) $t_H = H/h = 234.2$, $t_D = D_x/D_y = 0.11$; b) $t_H = 582.3$, $t_D = 9.42$

The dependence of non-dimensional critical force $\chi = F_{cr,cal}/K_c$ (correcting functions) upon $t_L = L/H$ and $t_B = B/H$ is presented in Fig. 4 for parameters $t_H = H/h = 234.2$, $t_D = D_x/D_y = 0.1$ (Fig. 4a) and $t_H = 582.3$, $t_D = 9.42$ (Fig. 4b). Increasing the dimensions of side walls of the box leads to the growth of non-dimensional compression force. This confirms the commonly known fact that the critical compression force increases with the growing perimeter of the box base. The studied intervals of the parameter changes had a non-linear character of growth. The following dependences occured: $F_{cr} \sim L^{0.122}$, $F_{cr} \sim B^{0.141}$, $F_{cr} \sim H^{-0.234}$, $F_{cr} \sim h^{1.971}$. An increase of the box height leads to a decrease of the critical compression force.

A comparison of the predicted forces (Table 2, Column 6) and the experimental critical forces (Table 2, Column 5) is shown in Fig. 5 (analysis similar to [15]). The line $F_{cr,calc} = F_{cr,exp}$ represents the calculated (Equation 16) critical forces and lines indicated +20% and -20% show the region where the absolute value of the relative error $\left|F^i_{cr,exp} - F^i_{cr,calc}\right| / F^i_{cr,exp} < 0.2$ and includes not less than 80% of obtained values.

The obtained expression of the critical box compression force gives a possibility to select appropriate box sizes, thickness and material of box walls.

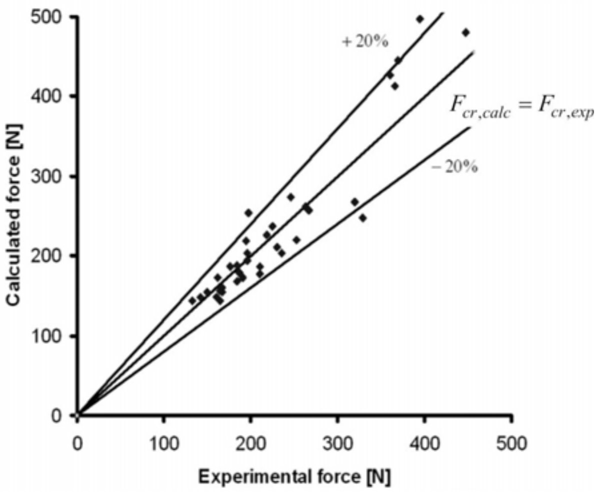

Fig. 5. Prediction of the critical compression force $F_{cr,cal}$ (16) for package compared with experimental data $F_{cr,exp}$

By writing the data from Table 2 into Equation (16), we calculated the values of the critical compression force. The results are presented in Table 2 (column 6). The average deviation of the calculated values $F^i_{cr,calc}$ from experimental $F^i_{cr,exp}$, $i = 1,\ldots,36$ determined by the following formula

$$\bar{e} = \frac{100}{n}\sum_{i=1}^{n}\frac{\left|F^i_{cr,exp} - F^i_{cr,calc}\right|}{F^i_{cr,exp}} \tag{18}$$

was $\bar{e} = 10.02\%$.

Conclusions

The new mathematical model of critical compression force was proposed and analyzed to describe the compression of paperboard package.

The model approximates the findings of experimental tests ($R^2 = 0.821$). This model can be also considered as accurate ($R^2 > 0.8$). Five unknown and constant values were determined from the experimental measurements. The average deviation of the calculated critical compression force from experimental was 10.02%.

The created engineering calculation procedure of rectangular - parallelpiped paperboard packages allows to optimize its strength characteristics including the package height.

The comparison of theoretical and experimental testing has showed a sufficient accuracy of the results.

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SYMBOLS:

BCT (N) – box compression strength;
D_x (N m) – rigidity in machine (x = *MD*) and cross-machine direction (x = *CD*);
D_y (N m) – rigidity in machine (y = *MD*) and cross-machine direction (y = *CD*);
h (m) – thickness;
$F_{cr,\,calc}$ (N) – critical compression force calculated from Eq. (16);
$F_{cr,\,exp}$ (N) – measured force (obtained experimentally);
K_c (N) – multiplier in (3);
LxBxH (m) – length x width x height;
SCT_x (N/m) – compressive strength in machine (x = *MD*) and cross-machine direction (x = *CD*);
SCT_y (N/m) – compressive strength in machine (y = *MD*) and cross-machine direction (y = *CD*);
x – direction of the compression load;
t_L ; t_B ; t_H ; t_D – non-dimensional parameters;
H (m) – package height decrease at critical compression force

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W 325 dni dookoła świata

Jacht SANADA wrócił do portu macierzystego w Gdyni

W sobotę 18. czerwca 2016 r. jacht SANADA zakończył uroczyście swój 325-dniowy rejs dookoła świata w porcie macierzystym w Gdyni. Dokładnie w samo południe żaglowiec wraz z załogą został uroczyście przywitany przez rodzinę i znajomych żeglarzy oraz przez członków Yacht Klubu Morskiego Columbus ze Świecia. Cumowaniu i powitaniu załogi towarzyszyła oprawa muzyczna w wykonaniu Miejskiej Orkiestry Dętej z Redy.

W trakcie 325-dniowego rejsu żaglowiec dowodzony przez kapitana Witolda Stempkowskiego przepłynął 37000 mil morskich (w tym 24000 mil dookoła świata), płynąc po trzech oceanach, dziewięciu morzach oraz brazylijskiej rzece Rio Paraiba. W rejsie uczestniczyło wymiennie 56 żeglarzy w wieku od 4 do 74 lat.

Warto podkreślić, iż kapitan Witold Stempkowski jest prezesem firmy TSC International związanej z przemysłem papierniczym, a podczas rejsu wykazał, że da się zarządzać firmą nawet z bezkresnych wód oceanów oraz jednocześnie realizować swoje marzenia.

Jacht Sanada, którego konstruktorem jest Juliusz Strawiński, został zbudowany w 2010 r. Jego nazwa pochodzi od imion córki i wnucząt kapitana. Sam kapitan Witold Stempkowski podkreśla, iż SANADA jest jednym z wielu jachtów zbudowanych i ochrzczonych w stoczni SM Europe na terenie Stoczni Gdańskiej, ale jej liczne podróże oraz związana z nimi działalność na rzecz branży papierniczej nadają jej wyjątkowy, nietypowy charakter.

Twórca wyprawy dookoła świata to przedsiębiorca i żeglarz Witold Stempkowski, który od 2011 r. na jachcie Sanada organizuje pod ogólną nazwą „Paper Ribbon" (Papierowa Wstęga) projekty łączące biznes z żeglarstwem.

W dniach 24-26 czerwca 2016 r., podczas Święta Morza w Gdyni można będzie nie tylko zwiedzać jacht, zacumowany w gdyńskiej marinie przy Basenie Zaruskiego, a także odbyć bezpłatny rejs Sanadą po Zatoce Gdańskiej. Szczegóły na www.swietomorza.eu.

Więcej informacji w następnym numerze „Przeglądu Papierniczego".

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